INVERSE BOUNDARY-VALUE PROBLEM OF A DRYING PROCESS

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The authors consider the problem of determination of the boundary moisture content and the nonstationary Biot number from the measured weight of an element. The algorithm is based on a solution of the corresponding direct problem in the form of a Duhamel integral.

A drying process is involved in many technologies of the food and chemical industries and the production of building materials. To calculate optimum drying regimes, it is necessary to know the moisture content field of a sample. In calculations, use is most often made of the mean moisture content and the moisture content at the center and on the surface of an element.

The relative mean moisture content of an element is easily determined experimentally from the tensometric curves of mass loss, but experimental determination of the surface moisture content is rather difficult. On the other hand, determination of the surface moisture content makes it possible to calculate the moisture content field in a sample using available methods of solution of direct problems in drying theory [1].

In practice, as a rule, the sample weight $G(\mathrm{Fo})$ is measured, which is proportional to the mean moisture content. Let us introduce the dimensionless mean moisture content

$$u_{\mathrm{m}}(\mathrm{Fo}) = \frac{G(0) - G(\mathrm{Fo})}{G(0)}.$$

The direct drying problem for a plane one-dimensional element, most often used in theory, has the following form in dimensionless variables with symmetric boundary conditions of the first kind:

$$\frac{\partial u}{\partial \mathrm{Fo}} = \frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le 1, \quad \frac{\partial u(0, \mathrm{Fo})}{\partial x} = 0; \quad u(x, 0) = 0; \quad u(1, \mathrm{Fo}) = \varphi(\mathrm{Fo}). \tag{1}$$

The mean moisture content of an element may be found from the exact solution of problem (1) in the space of Laplace transforms [2]:

$$u_{\mathrm{m}}(p) = \frac{\mathrm{th}\sqrt{p}}{\sqrt{p}}\varphi(p), \tag{2}$$

where $u_{\mathrm{m}}(p)$, $\varphi(p)$ are the Laplace transforms of the functions $u_{\mathrm{m}}(\mathrm{Fo})$, $\varphi(\mathrm{Fo})$; p is the parameter of a Laplace transformation.

Passing to the inverse transforms in (2), we obtain a solution in form of a Duhamel integral:

$$u_{\mathrm{m}}(\mathrm{Fo}) = \int_0^{\mathrm{Fo}} \Pi(\mathrm{Fo} - \tau)\varphi(\tau)\, d\tau, \tag{3}$$

where $\Pi(\mathrm{Fo}) \doteqdot \mathrm{th}\sqrt{p}/\sqrt{p}$.

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The inverse boundary-value problem consists in determining the surface moisture content $\varphi(\mathrm{Fo})$ from Eq. (3) using a dependence $u_m(\mathrm{Fo})$ known from experiment. Equation (3) is Volterra integral equation of the first kind, and such a problem is incorrectly stated [3, 4]. To solve incorrect problems, it is necessary to employ regularization methods; however, since the kernel of integral equation (3) has a rather complicated form, use of well known algorithms of solution of the formulated problem is extremely laborious and is unsuitable for engineering calculations [5].

Regularization methods allow one to solve a Volterra equation of the first kind both when $u_m(\mathrm{Fo})$ is specified empirically and when the kernel of integral equation (3) is specified incorrectly. The accuracy of the solution obtained in such cases depends, naturally, on the degree of inaccuracy of the kernel and the left-hand side of Eq. (3).

To obtain an approximate kernel of Eq. (3), we will use the method of imaginary frequencies [6]. However, the function th $\sqrt{p}/\sqrt{p}$ is inconvenient for obtaining a good approximation by this method, and therefore we first calculate the moisture content at the center of the element. In the space of Laplace transforms, instead of Eq. (2) we have

$$u(0, p) = \frac{\sqrt{p}}{\mathrm{sh}\sqrt{p}} u_m(p), \tag{4}$$

$$u(0, p) = \frac{1}{\mathrm{ch}\sqrt{p}} \varphi(p). \tag{5}$$

Passing to the inverse transform in Eq. (5), we obtain

$$u(0, \mathrm{Fo}) = \int_0^{\mathrm{Fo}} \Pi_1(\mathrm{Fo} - \tau)\varphi(\tau)\,d\tau. \tag{6}$$

in Eq. (6), the boundary moisture content $\varphi(\mathrm{Fo})$ is to be found, while the moisture content at the center of the element $u(0, \mathrm{Fo})$ may be easily determined using the standard heat conduction formulas for a sphere since the transfer functions of these processes coincide [2].

In Eq. (6), unlike (3), an approximation of the kernel may be sufficiently exactly obtained by approximating $1/\mathrm{ch}\sqrt{p}$ by a rational fractional function [7].

We now replace the kernel of integral equation (6) by the approximate $\tilde{\Pi}_1(\mathrm{Fo})$ and then perform regularization by the Lavrentiev method [8], transforming Eq. (6) into Volterra equation of the second kind:

$$u(0, \mathrm{Fo}) = \int_0^{\mathrm{Fo}} \tilde{\Pi}_1(\mathrm{Fo} - \tau)\varphi(\tau)\,d\tau + \alpha\varphi(\mathrm{Fo}), \tag{7}$$

where α is the regularization parameter.

Performing the Laplace transformation of Eq. (7), we arrive at

$$\varphi(p) = u(0, p)/(\tilde{\Pi}_1(p) + \alpha), \tag{8}$$

where $\tilde{\Pi}_1(p) = 1/(1 + 0.456p + 0.0209p^2)$.

Passing to the inverse transform in (8), we obtain the working formula

$$\varphi(\mathrm{Fo}) = \frac{1}{\alpha}\left[u(0, \mathrm{Fo}) - k\int_0^{\mathrm{Fo}} \exp[-\beta(\mathrm{Fo} - \tau)]\sin[\omega(\mathrm{Fo} - \tau)]\,u(0, \tau)\,d\tau\right], \tag{9}$$

where $\beta = 10.9091$; $k = 47.847/(\omega\alpha)$; $\omega = \sqrt{((1 + \alpha)/\alpha)\cdot 47.847 - \beta^2}$.

The regularization parameter α is determined from the following criterion [4]: $\|\varphi(\mathrm{Fo}, \alpha_{i+1}) - \varphi(\mathrm{Fo}, \alpha_i)\| = \min$, where $\alpha_{i+1} = \theta\alpha_i$, $0 < \theta < 1$.

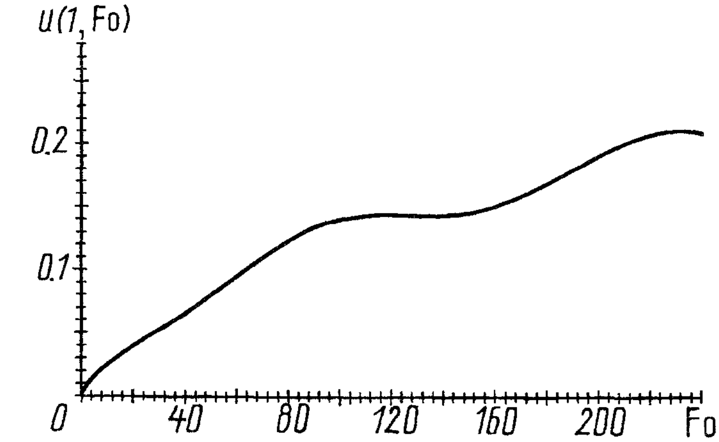

Fig. 1. Relative surface moisture content vs time.

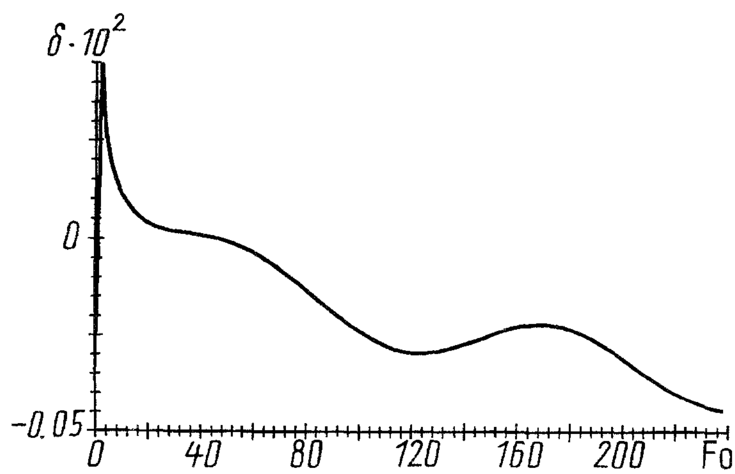

Fig. 2. Absolute error of reconstruction of the relative surface moisture content.

To check the accuracy of the suggested procedure, we solved a model problem. As initial data, we used approximated results of solution of the direct problem. A graph of the initial surface moisture content is shown in Fig. 1. The result of solution of the inverse problem by formula (9) was compared to the specified time dependence of the surface moisture content. In Fig. 2 the 100-fold magnified absolute error of reconstruction of the boundary moisture content is given.

Analysis of the results obtained shows that the accuracy of the suggested procedure is sufficient for engineering calculations. If necessary, the errors may be decreased by shortening the time interval between two measurements.

Reconstruction of the boundary moisture content also makes it possible to determine the law of change of the nonstationary Biot number in drying processes directly from the boundary conditions of the third kind:

$$\frac{\partial u(1, \mathrm{Fo})}{\partial x} = -\mathrm{Bi}(\mathrm{Fo})\,[u(1, \mathrm{Fo}) - u_b(\mathrm{Fo})], \tag{10}$$

where $u_b(\mathrm{Fo})$ is the ambient moisture content.

To determine Bi(Fo) from (10), we must know, in addition to the boundary moisture content $u(1, \mathrm{Fo})$, the mass flow at the boundary. For this, we are to find the derivative of the empirically specified function from the relation

$$\frac{\partial u(1, \mathrm{Fo})}{\partial x} = u'_{\mathrm{m}}(\mathrm{Fo}). \tag{11}$$

With account for relation (11), the equation for calculation of the nonstationary Biot number is

$$\mathrm{Bi}(\mathrm{Fo}) = \frac{u'_{\mathrm{m}}(\mathrm{Fo})}{u(1,\ \mathrm{Fo}) - u_b(\mathrm{Fo})}. \tag{12}$$

In conclusion, the suggested method of determination of the boundary moisture content from the measured mean moisture content together with any known method of determination of the derivative of the empirically specified function allows one to calculate the nonstationary Biot number by Eq. (12) and thus solve problems of intensification of drying processes. The simplicity of calculation formulas (9) and (12) makes it possible to develop software whose high speed and small required memory allow its use for creating monitoring and regulation systems for a drying process.

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